QT Compound & Inverse Functions

1. Given that $f(x) = x + 9$

(a) Find $f(6)$ (1 mark)

(b) Find $f(-2)$ (1 mark)

(c) Solve $f(x) = 12$ (2 marks)

(a) $f(6) = 6 + 9$
$= 15$.

(b) $f(-2) = -2 + 9$
$= 7$.

(c) $x + 9 = 12$
$x = 3$.

2. Given that $f(x) = 3x + 2$

(a) Find $f(3)$ (1 mark)

(b) Find $f(-4)$ (1 mark)

(c) Solve $f(x) = 14$ (2 marks)

(a) $f(3) = 3(3) + 2$
$= 9 + 2$
$= 11$.

(b) $f(-4) = 3(-4) + 2$
$= -12 + 2$
$= -10$

(c) $3x + 2 = 14$
$3x = 12$
$x = 4$.

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3. Given that $f(x) = 3x^2 + 4$

(a) Find $f(3)$ (2 marks)

(b) Find $f(-3)$ (2 marks)

(c) Solve $f(x) = 16$ (2 marks)

(a) $f(3) = 3(3)^2 + 4$
$= 3(9) + 4$
$= 31$.

(b) $f(-3) = 3(-3)^2 + 4$
$= 3(9) + 4$
$= 31$.

(c) $3x^2 + 4 = 16$
$3x^2 = 12$
$x^2 = 4$
$x = \pm 2$.

4. Given that $g(x) = x^2 + 5$

(a) Find $g(8)$ (1 mark)

(b) Find $g(-6)$ (1 mark)

(c) Work out the expression for $g^{-1}(x)$ (2 marks)

(d) Find $g^{-1}(x) = 4$ (2 marks)

(a) $g(8) = 8^2 + 5$.
$= 69$.

(b) $g(-6) = (-6)^2 + 5$.
$= 41$.

(c) $y = x^2 + 5$
$y - 5 = x^2$
$\sqrt{y-5} = x$
$g^{-1}(x) = \sqrt{x-5}$

(d) $g^{-1}(x) = 4$.
$\sqrt{x-5} = 4$
$x - 5 = 16$
$x = 21$.

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5. Given that $f(x) = 3x + 2$ and $g(x) = 2x - 6$

(a) Find $gf(3)$ (2 marks)

(b) Solve $f(x) = g(x)$ (2 marks)

(a) $f(3) = 3(3) + 2$

$= 11$

$gf(3) = 2(11) - 6$

$= 16$

(b) $3x + 2 = 2x - 6$

$x + 2 = -6$

$x = -8$

6. Given that $f(x) = 2x - 2$ and $g(x) = x + 3$

(a) Work out the expression for $f^{-1}(x)$ (2 marks)

(b) Work out the expression for $g^{-1}(x)$ (2 marks)

(c) Solve $f^{-1}(x) = g^{-1}(x)$ (2 marks)

(a) $y = 2x - 2$.

$y + 2 = 2x$

$\frac{y+2}{2} = x$

$f^{-1}(x) = \frac{x+2}{2}$

(b) $y = x + 3$

$y - 3 = x$

$g^{-1}(x) = x - 3$

(c) $\frac{x+2}{2} = x - 3$

$x + 2 = 2(x - 3)$

$x + 2 = 2x - 6$

$2 = x - 6$

$8 = x$

$\therefore x = 8$

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7. Given the function $f(x) = -5 - 9x$, find the value of $f^{-1}f(8)$ (3 marks)

$f(8) = -5 - 9(8)$

$= -77.$

$y = -5 - 9x$

$\frac{y+5}{-9} = x$

$\therefore f^{-1}(x) = \frac{x+5}{-9}.$

$f^{-1}f(8) = \frac{-77+5}{-9}$

$= 8$

8. A function f is defined such that $f(x) = \frac{x}{x-2}$

(a) Solve the equation $f(x) = \frac{5}{2}$ (2 marks)

(b) Find $f^{-1}(x)$

(a) $f(x) = \frac{\frac{5}{2}}{\frac{5}{2} - 2}$

$= \frac{\frac{5}{2}}{\frac{1}{2}}$

$= \frac{5}{2} \times \frac{2}{1}$

$= 5$

(b) $y = \frac{x}{x-2}$

$y(x-2) = x$

$xy - 2y = x$

$xy - x = 2y$

$x(y-1) = 2y$

$x = \frac{2y}{y-1}$

$f^{-1}(x) = \frac{2x}{x-1}$