Use black ink or ball-point pen.
Fill in the boxes at the top of this page with your name, centre number and candidate number.
Answer all questions.
Answer the questions in the spaces provided – there may be more space than you need.
Calculators may be used.

Information

1. The nth even number is $2n$.

The next even number after $2n$ is $2n + 2$

(a) Explain why.

Any even number is 2n (2×3=6 or 2×2=4)
Next even number is +2 (6+2=8 or 4+2=6)
(odd will be 2n+1)

(1)

(b) Write down an expression, in terms of n, for the next even number after $2n + 2$

$2n+4$

$(2n+2+2)$

$2n+4$

(1)

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

$2n + 2n+2 + 2n+4$

$6n+6$

$6(n+1)$

multiple of 6

(3)

(5 marks)

2. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n.

$(3n+1)(3n+1) - [(3n-1)(3n-1)]$

$9n^2 + 6n + 1 - (9n^2 - 6n + 1)$

$\cancel{9n^2} + 6n \cancel{+1} - \cancel{9n^2} + 6n \cancel{-1}$

$12n$

$4(3n)$

multiple of 4.

(3 marks)

3. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

Let n be the first whole number

$\therefore \quad \underset{\text{1st whole number}}{n} + \underset{\text{2nd whole number}}{(n+1)} = 2n + 1$

2n is a multiple of 2 and therefore even.
+1 must be odd as is one more than the even number.

(3 marks)

4. Prove that

$$(2n+3)^2 - (2n-3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

$(2n+3)(2n+3) - [(2n-3)(2n-3)]$

$4n^2+6n+6n+9 - [4n^2-6n-6n+9]$

$\cancel{4n^2}+12n\cancel{+9} - \cancel{4n^2}+12n\cancel{-9}$

$24n$

$8(3n)$

which is a multiple of 8

(3 marks)

*5. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

Difference between the squares

$= (n+1)^2 - n^2$

$= (n+1)(n+1) - n^2$

$= \cancel{n^2} + 2n + 1 - \cancel{n^2}$

$= \underline{2n+1}$

Sum of 2 consecutive numbers

$= n + n + 1$

$= \underline{2n+1}$

$\therefore$ Both are equal

(4 marks)

6. Prove that $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 5, for all positive integer values of n.

$$(5n+1)(5n+1) - [(5n-1)(5n-1)]$$

$$25n^2 + 10n + 1 - [25n^2 - 10n + 1]$$

$$\cancel{25n^2} + 10n \cancel{+1} \cancel{-25n^2} + 10n \cancel{-1}$$

$$20n$$

$$5(4n)$$

which is a multiple of 5.

(3 marks)

7. If $2n$ is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

Two consecutive even numbers are $2n$ and $2n+2$.

Sum of their squares $= (2n)^2 + (2n+2)^2$

$= 4n^2 + (2n+2)(2n+2)$

$= 4n^2 + 4n^2 + 8n + 4$

$= 8n^2 + 8n + 4$

$= 4(2n^2 + 2n + 1)$

multiple of 4.

(3 marks)

8. Prove that

$(n+1)^2 - (n-1)^2 + 1$ is always odd for all positive integer values of n.

$$(n+1)(n+1) - \left[(n-1)(n-1)\right] + 1$$

$$n^2 + 2n + 1 - \left[n^2 - 2n + 1\right] + 1$$

$$\cancel{n^2} + 2n + \cancel{1} \cancel{- n^2} + 2n \cancel{- 1} + 1$$

$$4n + 1$$

4n must be even ∴ +1 will be odd for any positive value of n.

(3 marks)

9. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

$$(n^2) + (n+1)^2$$

$$n^2 + (n+1)(n+1)$$

$$n^2 + n^2 + 2n + 1$$

$$2n^2 + 2n + 1$$

$$2n(n+1) + 1$$

$n(n+1)$ is 2 consecutive numbers multiplied together. The answer must always be even

[redacted] $6(6+1) = 42$

$2n(n+1)$ 2 × any even number is a multiple of 4 $2 \times 42 = 84$

So $2n(n+1) \underline{+1}$ is a multiple of 4 and will leave a remainder of 1 when divided by 4.

(4 marks)