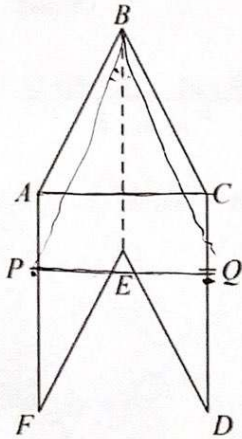




## QT Tough Questions - Proving Cosine



The diagram shows a hexagon ABCDEF.

ABEF and CBED are congruent parallelograms where

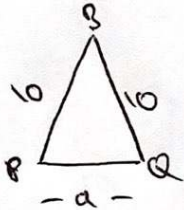
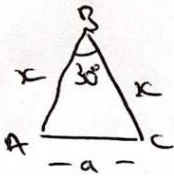
$AB = BC = x$  cm.

P is the point on AF and Q is the point on CD such that

$BP = BQ = 10$  cm.

Given that angle  $ABC = 30^\circ$ , prove that

$$\cos PBQ = 1 - \frac{(2-\sqrt{3})x^2}{200}$$



$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= x^2 + x^2 - 2(x)(x)\left(\frac{\sqrt{3}}{2}\right) \end{aligned}$$

$$a^2 = 2x^2 - x^2\sqrt{3}$$

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= 10^2 + 10^2 - 2(10)(10)(\cos A) \end{aligned}$$

$$a^2 = 200 - 200 \cos A$$

$$2x^2 - x^2\sqrt{3} = 200 - 200 \cos A$$

$$200 \cos A = 200 - 2x^2 + x^2\sqrt{3}$$

$$200 \cos A = 200 - x^2(2 - \sqrt{3})$$

$$\cos A = \frac{200 - x^2(2 - \sqrt{3})}{200}$$

$$\cos A = \frac{200 - x^2(2 - \sqrt{3})}{200}$$

$$\cos A = \frac{200}{200} - \frac{x^2(2 - \sqrt{3})}{200}$$

$$\cos A = 1 - \frac{(2 - \sqrt{3})x^2}{200}$$

$$A = \underline{\underline{\text{Angle PSQ}}}$$

