QT Algebraic Proof

1. Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6.

$2n + 2n + 2 + 2n + 4$

$6n + 6$

$6(n + 1)$

factor ∴ multiple

2. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n.

$(3n+1)(3n+1) - [(3n-1)(3n-1)]$

$9n^2 + 6n + 1 - [(9n^2 - 6n + 1)]$

$\cancel{9n^2} + 6n \cancel{+1} \cancel{-9n^2} + 6n \cancel{-1}$

$12n = 4(3n)$

factor ∴ multiple

3. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

Diff in squares = sum

$(n+1)^2 - (n)^2 = n + (n+1)$

$(n+1)(n+1) - n^2 = 2n + 1$

$\cancel{n^2} + 2n + 1 \cancel{-n^2} = 2n + 1$

$2n + 1 = 2n + 1$

Diff in squares equals sum

4. Prove algebraically that the sum of the squares of any 2 odd positive integers is always even.

$2x+1$ = 1st odd $2y+1$ = 2nd odd

$(2x+1)^2 + (2y+1)^2$

$(2x+1)(2x+1) + (2y+1)(2y+1)$

$4x^2+4x+1 + 4y^2+4y+1$

$4x^2+4x+4y^2+4y+2$

$2(2x^2+2x+2y^2+2y+1)$

factor ∴ multiple

5. 5(x - c) = 4x - 5 where c is an integer. Prove that x is a multiple of 5.

$5x - 5c = 4x - 5$

$-4x$ $-4x$

$x - 5c = -5$

$+5c$ $+5c$

$x = 5c - 5$

$x = 5(c-1)$

factor ∴ multiple