QT Completing the Square

1. (a) Write $x^2 - 6x + 1$ in the form $(x + a)^2 + b$ where a and b are integers

(2 marks)

$(x-3)^2 - 9 + 1$

$(x-3)^2 - 8$

$(x-3)(x-3)$

$x^2 - 6x + 9$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = x^2 - 6x + 1$ $(3, -8)$ (1 mark)

2. (a) Write $x^2 + 10x + 8$ in the form $(x + a)^2 + b$ where a and b are integers

(2 marks)

$(x+5)^2 - 25 + 8$

$(x+5)^2 - 17$

$(x+5)(x+5)$

$x^2 + 10x + 25$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = x^2 + 10x + 8$ (1 mark)

$(-5, -17)$

QT Completing the Square

3. (a) Write $x^2 + 3x - 7$ in the form $(x + a)^2 + b$ where a and b are integers

(2 marks)

$\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} - \frac{28}{4}$

$\left(x+\frac{3}{2}\right)^2 - \frac{37}{4}$

$\left(x+\frac{3}{2}\right)\left(x+\frac{3}{2}\right)$

$x^2 + \frac{6}{2}x + \frac{9}{4}$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = x^2 + 3x - 7$ (1 mark)

$\left(-\frac{3}{2}, \frac{-37}{4}\right)$ or $(-1.5, -9.25)$

4. (a) Write $x^2 - 2x - 6$ in the form $(x + a)^2 + b$ where a and b are integers

(2 marks)

$(x-1)^2 - 1 - 6$

$(x-1)^2 - 7$

$(x-1)(x-1)$

$x^2 - 2x + 1$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = x^2 - 2x - 6$ (1 mark)

$(1, -7)$

QT Completing the Square

5. By completing the square, find the coordinates of the turning point of the curve with the equation $y = x^2 + 10x - 8$. You must show all your working.

(3 marks)

$(x+5)^2 - 25 - 8$

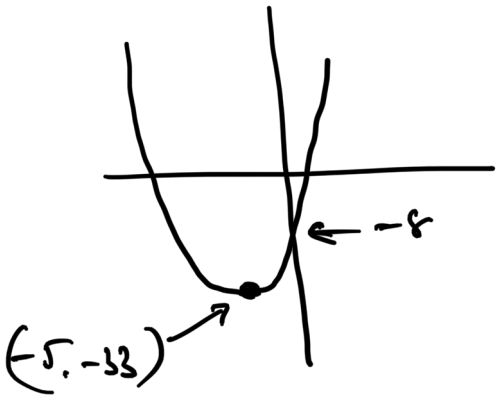
$(x+5)^2 - 33$

Turning Point

$(-5, -33)$

$(x+5)(x+5)$

$x^2 + 10x + 25$



6. By completing the square, find the coordinates of the turning point of the curve with the equation $y = x^2 - 6x + 2$. You must show all your working.

(3 marks)

$(x-3)^2 - 9 + 2$

$(x-3)^2 - 7$

Turning point $(3, -7)$

$(x-3)(x-3)$

$x^2 - 6x + 9$

QT Completing the Square

7. By completing the square, find the coordinates of the turning point of the curve with the equation $y = x^2 - 5x + 1$. You must show all your working.

(3 marks)

$(x - \frac{5}{2})^2 - \frac{25}{4} + \frac{4}{4}$

$(x - \frac{5}{2})^2 - \frac{21}{4}$

Turning point

$(\frac{5}{2}, -\frac{21}{4})$ or $(2.5, -5.25)$

$(x - \frac{5}{2})(x - \frac{5}{2})$

$x^2 - \frac{10}{2}x + \frac{25}{4}$

$-5x$

8. By completing the square, find the coordinates of the turning point of the curve with the equation $y = x^2 + 0.5x + 7$. You must show all your working.

(3 marks)

$(x + \frac{1}{4})^2 - \frac{1}{16} + 7$

$(x + \frac{1}{4})^2 - \frac{1}{16} + \frac{112}{16}$

$(x + \frac{1}{4})^2 + \frac{111}{16}$

Turning point

$(-\frac{1}{4}, \frac{111}{16})$ or $(-0.25, 6.9375)$

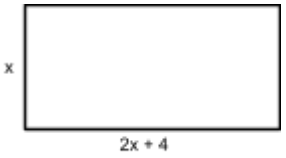
$(x + \frac{1}{4})(x + \frac{1}{4})$

$x^2 + \frac{2}{4}x + \frac{1}{16}$

$\frac{1}{2}$ or 0.5

QT Completing the Square

9. A rectangle has sides of $x\ cm$ and $(2x\ +\ 4)\ cm$ as shown.
The area of the rectangle is $30\ cm^2$.



(a) Show that $(x\ +\ 1)^2\ -\ 16\ =\ 0$ (3 marks)

$x(2x+4) = 30$

$2x^2+4x = 30$

$2x^2+4x-30 = 0$

$x^2+2x-15 = 0$

$(x+1)^2-1-15 = 0$

$\therefore (x+1)^2-16 = 0$

$(x+1)(x+1)$

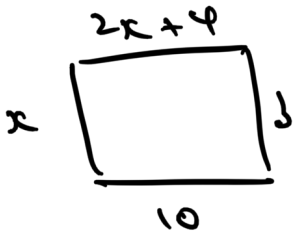
x^2+2x+1

(b) Hence, or otherwise, find the perimeter of the rectangle (2 mark)

① $x^2+2x-15$

$(x+5)(x-3)$

$\therefore x = -5$ or $x = 3$



$\therefore$ Perimeter $= 26\ cm$.

② $(x+1)^2-16 = 0$

$(x+1)^2 = 16$

$x+1 = \pm 4$

$x = -1+4 = 3$

$x = -1-4 = -5$

$\therefore$ Perimeter $= 26\ cm$

QT Completing the Square

10. (a) Write $2x^2 - 12x + 24$ in the form $a(x + b)^2 + c$ where a, b and c are integers (3 marks)

$2[(x^2 - 6x + 12)]$

$2[(x-3)^2 - 9 + 12]$

$2[(x-3)^2 + 3]$

$2(x-3)^2 + 6$

$(x-3)(x-3)$

$x^2 - 6x + 9$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 2x^2 - 12x + 24$ (1 mark)

Turning point $(3, 6)$

11. (a) Write $2x^2 + 8x + 10$ in the form $a(x + b)^2 + c$ where a, b and c are integers (3 marks)

$2[(x^2 + 4x + 5)]$

$2[(x+2)^2 - 4 + 5]$

$2[(x+2)^2 + 1]$

$2(x+2)^2 + 2$

$(x+2)(x+2)$

$x^2 + 4x + 4$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 2x^2 + 8x + 10$ (1 mark)

Turning point $(-2, 2)$

QT Completing the Square

12.(a) Write $3x^2 + 6x - 8$ in the form $a(x+b)^2 + c$ where a, b and c are integers (3 marks)

$3\left[\left(x^2 + 2x - \frac{8}{3}\right)\right]$

$3\left[(x+1)^2 - 1 - \frac{8}{3}\right]$

$3\left[(x+1)^2 - \frac{11}{3}\right]$

$3(x+1)^2 - 11$

$(x+1)(x+1)$

$x^2 + 2x + 1$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 3x^2 + 6x - 8$ (1 mark)

Turning point $(-1, -11)$

13. (a) Write $4x^2 - 8x - 7$ in the form $a(x+b)^2 + c$ where a and b are integers (3 marks)

$4\left[\left(x^2 - 2x - \frac{7}{4}\right)\right]$

$4\left[(x-1)^2 - 1 - \frac{7}{4}\right]$

$4\left[(x-1)^2 - \frac{11}{4}\right]$

$4(x-1)^2 - 11$

$(x-1)(x-1)$

$x^2 - 2x + 1$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph

$y = 4x^2 - 8x - 7$ (1 mark)

Turning point $(1, -11)$

QT Completing the Square

NON CALCULATOR

14. By completing the square, solve $x^2 = 22x - 5$

Give your answers in surd form. (5 marks)

$x^2 - 22x + 5 = 0$

$(x-11)^2 - 121 + 5 = 0$

$(x-11)^2 - 116 = 0$

$(x-11)^2 = 116$

$x - 11 = \pm\sqrt{116}$

$x = 11 + \sqrt{116}$ or $11 + 2\sqrt{29}$

$= 11 - \sqrt{116}$ $11 - 2\sqrt{29}$

$(x-11)(x-11)$

$x^2 - 22x + 121$

15. By completing the square, solve $x^2 + 5x + \frac{17}{4} = 0$

Give your answers in surd form. (5 marks)

$\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{17}{4} = 0$

$\left(x + \frac{5}{2}\right)^2 - \frac{8}{4} = 0$

$\left(x + \frac{5}{2}\right)^2 = 2$

$x + \frac{5}{2} = \pm\sqrt{2}$

$x = -\frac{5}{2} + \sqrt{2}$ or $-\frac{5}{2} - \sqrt{2}$

$\left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right)$

$x^2 + \frac{10}{2}x + \frac{25}{4}$

$5x$