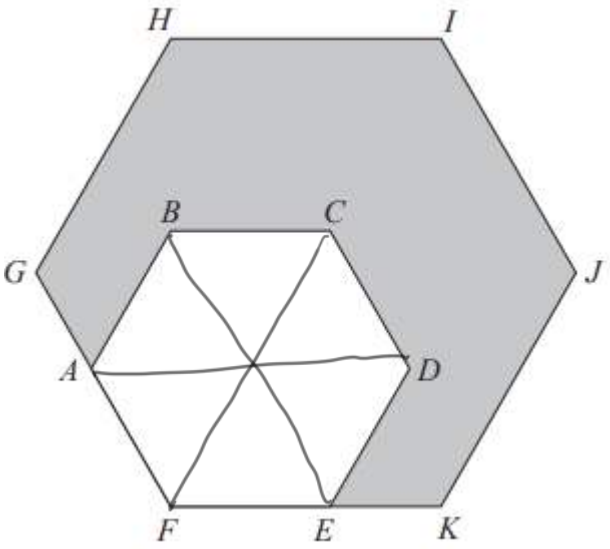
5 hard questions - part 2

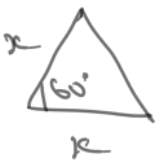
1. ABCDEF is a regular hexagon with sides of length x.
This hexagon is enlarged, centre F, by scale factor 'p' to give hexagon FGHIJK.

Show that the area of the shaded region in the diagram is given by $\frac{3\sqrt{3}}{2}(p^2 - 1)x^2$

(4 marks)



Area ABCDEF



$\frac{1}{2}ab\sin C$

$\frac{1}{2} \cdot x \cdot x \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4}x^2$

Total $= \frac{6\sqrt{3}}{4}x^2 = \frac{3\sqrt{3}}{2}x^2$

Enlarged FGHIJK $= \frac{3\sqrt{3}}{2}(px)^2$

Scaled $\frac{3\sqrt{3}}{2}p^2x^2 - \frac{3\sqrt{3}}{2}x^2$

$\frac{3\sqrt{3}}{2}x^2(p^2-1)$ or $\frac{3\sqrt{3}}{2}(p^2-1)x^2$

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2. $6x^2 = 7xy + 20y^2$ where x > 0 and y > 0

Find the ratio x : y (4 marks)

$6x^2 - 7xy - 20y^2 = 0$

-120

$-15 \quad +8$

$6x^2 - 15xy + 8xy - 20y^2 = 0$

$3x(2x - 5y) + 4y(2x - 5y) = 0$

$(3x + 4y)(2x - 5y) = 0$

$3x + 4y = 0$

$3x = -4y$

$x : y$

$-4 : 3$

$2x - 5y = 0$

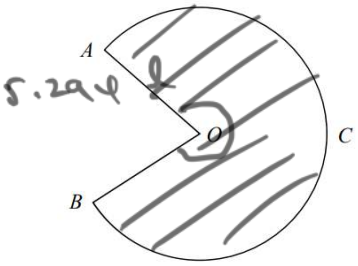
$2x = 5y$

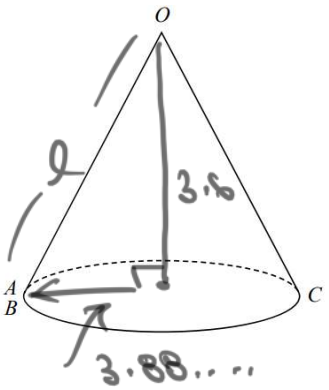
$x : y$

$5 : 2$

5 hard questions - part 2

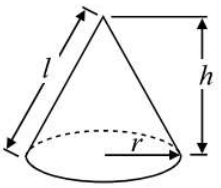
3. The diagram shows a sector OACB of a circle with centre O. (5 marks)
The point C is the midpoint of the arc AB.
The diagram also shows a hollow cone with vertex O.
The cone is formed by joining OA and OB.





Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl



The cone has volume $56.8cm^3$ and height $3.6cm$
Calculate the size of angle AOB of sector OACB.
Give your answer correct to 3 significant figures.

$V = \frac{1}{3}\pi r^2 h$

$56.8 = \frac{1}{3}\pi r^2 (3.6)$

$\frac{56.8}{1.2\pi} = r^2$

$3.88158 = r$ (1)

Use Pythag.

$l^2 = (3.88158)^2 + 3.6^2$

$l = 5.294$ (2)

Surface Area

$\pi r l = \pi (3.88158)(5.294)$

$= 64.557 cm^2$ (3)

$\frac{\angle AOB}{360} \times (\pi \times 5.294^2) = 64.557$

$\frac{\angle AOB}{360} = 0.73326$

$\therefore$ Angle AOB = 264°

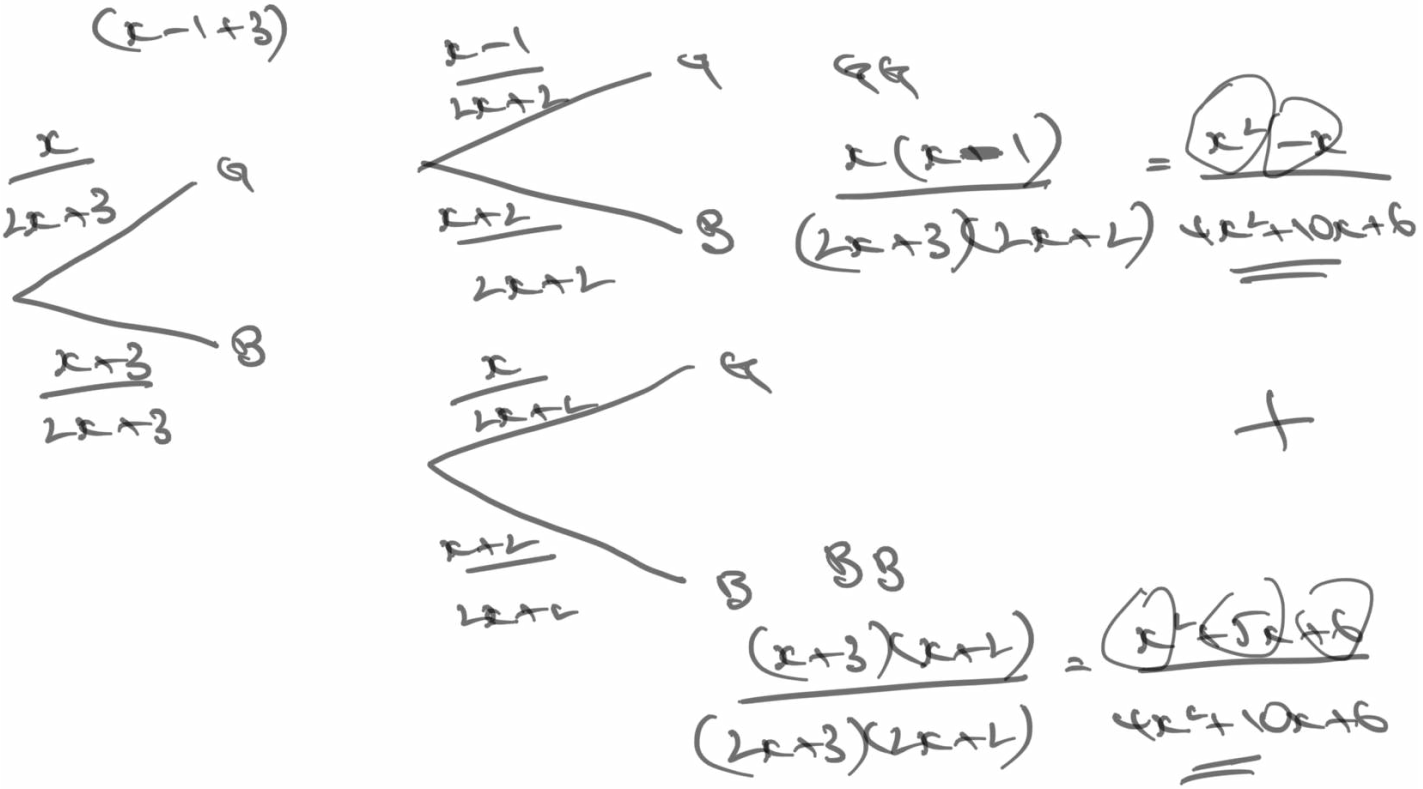
5 hard questions - part 2

4. There are only green pens and blue pens in a box.
There are three more blue pens than green pens in the box.
There are more than 12 pens in the box.
Simon is going to take at random two pens from the box.

The probability that Simon will take two pens of the same colour is $\frac{27}{55}$

Work out the number of green pens in the box (5 mark)

G = x
B = x + 3
T = 2x + 3



$\frac{2x^2 + 4x + 6}{4x^2 + 10x + 6} \times \frac{27}{55}$

$110x^2 + 220x + 330 = 108x^2 + 270x + 162$

$2x^2 - 50x + 168 = 0$

$x^2 - 25x + 84 = 0$

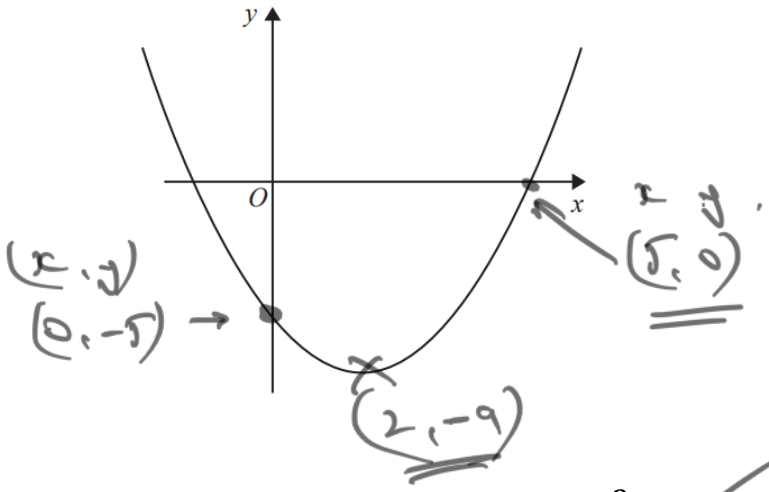
$(x - 21)(x - 4) = 0$

$x = 21$ or $x = 4$

Green pens = 21

5 hard questions - part 2

5. Here is a sketch of a curve.



The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.
The points (0, -5) and (5, 0) lie on the curve.

Find the coordinates of the turning point of the curve. (4 marks)